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Adverse Selection, Contract Design, and Investment Distortion*

THOMAS H. NOE AND MICHAEL J. REBELLO

Department of Finance, College of Business Administration, Georgia State University, Atlanta, Georgia 30303

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We examine the design of compensation contracts and determination of investment policies when a manager has private information regarding the effect of investment on both the firm's cash flows and the private benefits she is able to extract from employment. We show that, in general, the optimal mechanism is characterized by a menu of salary and option contracts. When the manager's private information relates only to the firm's cash flows, the firm overinvests relative to the Pareto optimal level. On the other hand, if the private information relates only to private benefits, the firm will underinvest. *Journal of Economic Literature* Classification Numbers: G31, D82.

1. INTRODUCTION

Most of the considerable research on managerial incentives and corporate investments has been conducted in the context of models in which the determination of the optimal contractual form is not a focal point of the analysis (see, for example, Hirshleifer and Thakor, 1992; Shleifer and Vishny, 1989; Harris and Raviv, 1989). Likewise, the extensive literature on contract design in the presence of adverse selection and moral hazard (see, for example, Diamond, 1984; Gale and Hellwig, 1985; Innes, 1990; Townsend, 1979) has largely ignored the determination of investment expenditure. As both corporate investment and management compensa-

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tion contracts are endogenously determined, integrating the analysis of contract design with that of optimal investment expenditure provides additional insights into the relationship between investment and compensation (see, for example, Heckerman, 1975; Holmstrom and Weiss, 1985; Shah and Thakor, 1988).

In this paper, we provide insights into the relationship between investment and compensation by incorporating into the analysis limited liability and investment-related private benefits obtained by managers. In our framework, a firm is faced with the problem of determining a capital budget and designing a contract, satisfying limited liability, for a manager who has private information regarding future prospects. The manager could receive one of two private signals: G (good) or B (bad). Both private benefits and output are higher under signal G than under signal B.[1]

The firm can make a more efficient investment decision by using the manager's private information when determining its capital budget. To obtain this information, it has to communicate with the manager. In general, there exists a variety of mechanisms through which the manager and firm can communicate. Because their interests are not perfectly aligned, the manager may not have the incentive to accurately reveal her private information. To characterize the equilibrium allocations resulting from communication between the firm and the manager, we use the Revelation Principle (see Myerson, 1981; Harris and Townsend, 1981).

Since Holmstrom (1979), it has been recognized that optimal solutions to incentive problems maximize the sensitivity of contracts to the agent's information. In our framework, this contract design principle translates into selecting the contract–capital budget pair that minimizes misreporting incentives. Despite the complexity of the problem, the optimal contracts have a simple functional form, and the firm follows an equally simple rule for assigning contracts based on managerial reports. Whenever the manager reports G, she obtains an option on the firm's cash flows, and if she reports B, she is compensated using a salary contract.[2]

While the functional forms of the optimal contracts are independent of corporate investment policy, the investment policy is dependent on both the optimal contract design and the focus of the manager's private information. When the manager's private information relates primarily to firm output, mitigation of the incentive problem leads the firm toward overinvestment relative to the Pareto optima level. This occurs because increased investment reduces the impact of differential managerial informa-

[1] This has a simple human capital interpretation—when productivity is high, so is the value of the manager's human capital, her private benefit from employment.

[2] These contracts are idealized versions of observed salary and option contracts. However, we use the terms option and salary contracts to avoid having to invent new terminology.

tion on the value of the salary contract. Consequently, it reduces the manager's misreporting incentive if she receives the signal G. Increased investment also makes the value of the option contract more sensitive to differences in the manager's information, reducing the misrepresentation incentives of the manager if she receives the signal B.[3]

When the manager's private information relates primarily to private benefits, she will be willing to accept lower monetary compensation if she receives a more favorable signal. To prevent the firm from divining her true reservation compensation, the manager will have an incentive to underreport the size of her investment-related private benefit. When the signal G is associated with higher marginal private benefits, the most efficient way for the firm to eliminate this incentive for misrepresentation is to lower the investment level associated with the report of lower private benefits. In summary, when the manager's private information relates only to firm output, the solution to the incentive problem calls for overinvestment, while underinvestment may result when the manager's private information relates only to private benefits. However, when the manager's private information relates to firm output as well as private benefit generation, the direction of distortion in investment depends on which of the two factors is the primary constituent of the manager's private information.

Our results relate to Holmstrom and Weiss (1985) and Shah and Thakor (1988), who also examine the issue of contracting and investment with managerial private information regarding firm output. In Holmstrom and Weiss (1985) future output is known to the manager with certainty; i.e., there is no residual uncertainty. The optimal contractual form, regardless of the private signal, provides the manager with a payoff only if a prespecified output level is attained. In addition to depending on the level of investment and the state of the world, output depends on the manager's unobservable effort decision. Because the firm cannot disentangle the effects of low effort and "bad" states of the world, the manager has an incentive to report falsely that the bad state has occurred in order to justify high compensation despite low output. To mitigate this misreporting incentive, the second-best contract calls for reducing the level of investment below the Pareto optimal level when the bad state is reported. Shah and Thakor (1988) also establish the optimality of underinvestment

[3] The incentive to overinvest in order to lower the information sensitivity of the salary contract would not exist in the absence of limited liability, as in this case the firm could always offer the manager a fixed payment independent of the cash flows and level of investment in response to the report B. As the value of the fixed payment contract is insensitive to the manager's private information, such a contract would eliminate the incentive for the firm to distort investment. Even in the absence of limited liability, the determination of the optimal contract associated with the report G is not a trivial exercise.

in the context of a model with uncertain output, linear contracts, but no moral hazard problem.[4] They demonstrate that if the principal is relatively risk averse, efficient risk sharing dictates that the manager be given an output-sensitive contract. However, an output-sensitive contract provides the manager with the incentive to falsely report the state in order to obtain a larger share of the output. Thus, the firm underinvests to mitigate misreporting incentives.

These results are consistent with ours, which demonstrate that underinvestments in response to the report B may be optimal when the manager's private information relates only to private benefits. On the other hand, when managerial private information relates primarily to cash flows, we show that minimization of misreporting incentives calls for overinvestment. This result has no analog in either Holmstrom and Weiss (1985) or Shah and Thakor (1988) and depends on the interaction between investment level, residual uncertainty, and the report-contingent differences in the functional forms of the managerial compensation contracts. These trade-offs are absent in settings in which compensation takes the form of simple output quotas as in Holmstrom and Weiss (1985) or takes the form of linear sharing rules as is the case in Shah and Thakor (1988).

The remainder of the paper is organized as follows: In Section 2, we present our model. Section 3 contains a complete characterization of the firm's contract design problem. Section 4 is devoted to an examination of the firm's investment decision. Concluding remarks, along with empirical implications of our model, are presented in Section 5. Proofs of propositions are presented in the Appendix.

2. THE REVELATION PROBLEM

Consider a single-period, two-date setting in which all agents, including the owners of firms, are risk neutral and the risk-free rate of return is zero. At the beginning of the period, a firm has to determine the level of investment, I, in a project, where $I \in [0, \bar{I}]$. To undertake the project, it has to communicate and contract with a manager who possesses private information. First, the manager reports her information to the firm. Next, based on this report, the firm picks the cash flow sharing rule and the investment level. These choices must provide the manager with at least her reservation utility level, U_0.[5] The cash flows and private benefits generated by the investment are realized at the end of the period. At this

[4] See also Heckerman (1975).

[5] As becomes apparent from the following analysis, it is easy to incorporate signal-dependent reservation utility levels into our analysis through the private benefit component of the manager's utility.

time, the cash flows are divided between the manager and firm owner in accordance with the cash flow sharing rule.

The firm believes that the manager it is contracting with could have observed a good signal (G) or a bad signal (B), with prior probabilities μ and $1 - \mu$, respectively. The observed signal and the investment level jointly determine the firm's end-of-period cash flows and the private benefits obtained by the manager. The cash flows are represented by $\tilde{X}$, where $\tilde{X}(I, t) \equiv k(I)\tilde{Z}_t$, $t = G, B$.[6] The function $k(\cdot)$ is nonnegative, twice differentiable, increasing, and strictly concave, with $k'(0) = \infty$ and $k'(\bar{I}) = 0$. The term $\tilde{Z}_t$ represents the disturbance as a function of the manager's signal. The cumulative distribution function of $\tilde{Z}_t$ is given by $F_t(z)$, which is twice continuously differentiable with $F_t(0) = 0$. For each t, the support of $F_t(z)$ is contained in the interval $[0, M]$, where $M < \infty$. The distribution of the firm's cash flows is represented by $N(I, t, x)$, where

$$N(I, t, x) \equiv F_t\left[\frac{x}{k(I)}\right]. \tag{1}$$

The above assumptions imply that, for each t, the map $I \to N(I, t, x)$ is continuous when the distribution functions are topologized using the weak topology.

To arrive at a tractable solution to the revelation problem, we impose structure on the relationship between the cash flow distributions of the two types (signals). Define $\bar{F}_t(z) \equiv 1 - F_t(z)$, and let the ratio $r(z) = \bar{F}_B(z)/\bar{F}_G(z)$. If $r(x)$ is nonincreasing (decreasing) whenever $\bar{F}_G(z) \neq 0$, the disturbance terms $\tilde{Z}_G$ and $\tilde{Z}_B$ are (strictly) hazard rate ordered.[7] The motivation for this definition follows from differentiating $r(x)$. On differentiation, the derivative of this ratio, $r'(z) = r(z)[\eta_G(z) - \eta_B(z)] \leq 0$, where $\eta_t(z) \equiv f_t(z)/\bar{F}_t(z)$, the hazard rate for type t. It follows that requiring $r(z)$ to be nonincreasing is equivalent to the assumption that disturbance terms are hazard-rate-ordered (HRO); i.e., $\eta_G(z) \leq \eta_B(z)$, where $\min\{\bar{F}_G(z), \bar{F}_B(z)\} > 0$.[8] In the sequel, we always assume that the disturbance terms are HRO. In some cases, to obtain sharper characterizations, we explicitly impose strict HRO on the disturbance terms.

[6] In Section 5, we discuss the consequences of changing the assumption relating investment to output uncertainty.

[7] This assumption does not require that the disturbance terms have identical supports. It does however, require that both the lower and upper endpoints of the support for F_G be no lower than the lower and upper endpoints of the support for F_B, respectively.

[8] Because the term hazard rate is already well established in the statistics literature (see, for example, Ross, 1983, Chap. 8), for ease of exposition we refer to this restriction on the disturbance terms as the hazard-rate-ordering assumption (HRO). The hazard-rate ordering is also employed extensively in the economics literature (see, for example, Grossman and Hart, 1982).

The hazard-rate-ordering assumption implies first-order stochastic dominance (FSD) and is weaker than the commonly employed monotone likelihood ratio ordering. It can also be interpreted as requiring that FSD hold for every truncation of the distributions. Note that if the disturbance term $\tilde{Z}_B$ has an increasing hazard rate, HRO obtains if $\tilde{Z}_G$ is a positive shift of $\tilde{Z}_B$ (i.e., $\tilde{Z}_G = \tilde{Z}_B + \gamma, \gamma > 0$), or $\tilde{Z}_G$ is a multiplicative scaling of $\tilde{Z}_B$ with a scale factor greater than 1 (i.e., $\tilde{Z}_G = \gamma \tilde{Z}_B, \gamma > 1$). Let $\overline{N}(\cdot) = 1 - N(\cdot)$, where $N(\cdot)$ is defined in (1). Our restrictions on the distribution terms, $F_t(\cdot)$, imply that

$$\frac{\overline{N}(I, B, z)}{\overline{N}(I, G, z)}$$

is nonincreasing in z.

The manager's private benefits as a function of the firm's investment expenditure are represented by $p(I, t)$. This function is twice continuously differentiable, nondecreasing, and concave in I. Note that $p(I, t)$ can be interpreted as the expected value of the end-of-period private benefits, which could be a function of the realized cash flow, $\tilde{X}$. Let $D_I p(I, t)$ be the derivative of $p(I, t)$ with respect to I, and, to simplify the analysis, let $D_I p(\bar{I}, t) = 0$. To capture the notion that the manager obtains larger total benefits if she receives the signal G, we assume that $p(I, G) \geq p(I, B)$ for all $I \in [0, \bar{I}]$.

To ensure that the firm will always be able to design a compensation scheme which will yield a positive profit, assume that

$$\int_0^\infty x\, N(I, t, dx) + p(\bar{I}, t) > U_0 + \bar{I}, \qquad \text{for } t = G, B. \tag{F}$$

To ensure that the manager is provided with some of the cash flows for her participation constraint to be satisfied, let $U_0 > p(\bar{I}, G)$.

By providing the manager with a contract, the firm determines her share of the cash flows as well as the residual cash flows which it obtains. Specifically, $s(x)$ and $c(x)$ represent the manager's and the firm's share of the cash flows, respectively. The firm's and the manager's shares are increasing functions of the cash flows and satisfy limited liability; i.e., $s(x)$ and $c(x)$ are maps from $[0, k(\bar{I})M]$ to itself, satisfying conditions

$$s(x) + c(x) = x,$$

$$s(x) \geq 0, \qquad c(x) \geq 0, \tag{LL}$$

$$s(x) \text{ and } c(x) \text{ are nondecreasing.} \tag{M}$$

The set of contracts which satisfy these conditions, $\mathcal{S}$, is rich and includes salary contracts, stock ownership contracts, and option contracts. The limited liability constraint (LL) on contract payoffs is realistic, as it is an important restriction on contracting by firms. The requirement that contracts satisfy the monotonicity condition (M) is commonly used in the literature.[9] It can be justified by assuming that, if they so desire, both the firm and the manager are able to choose stochastically dominated production technologies. Therefore, if their payoffs are not monotone increasing in the cash flows, they will have an incentive to sabotage the firm by substituting inferior production technologies.[10]

These restrictions on the cash flow sharing rules ensure that $s(x) - s(y) \geq 0$ if $x - y \geq 0$. The facts that $c(x)$ is increasing and $c(x) = x - s(x)$ imply that $s(x) - s(y) \leq x - y$. It follows that

$$|s(x) - s(y)| \leq |x - y|.$$

Therefore, the sharing rules are Lipshitz and, thus, are absolutely continuous (see Royden, 1968, p. 108). The above expression, combined with the fact that $s(\cdot)$ and $c(\cdot)$ are bounded, implies that $\mathcal{S}$ is a compact set in the supremum–norm topology (see Royden, 1968, problem 41, p. 180). Because $s(\cdot)$ is Lipschitz, there exists a function $w(x) \equiv s'(x)$ almost everywhere on the interval $[0, k(\bar{I})M]$, such that

$$s(x) = \int_0^x w(r)\, dr.^{11} \tag{2}$$

Further, as $s'(x)$ always lies in the interval $[0, 1]$, $w(x)$ can be selected so as to always lie in the same interval.

Let the payoff to a manager of type t, given the sharing rule–investment pair (s, I), be

$$U^t(I, s) \equiv \int_0^\infty sN(I, t, dx) + p(I, t).$$

The first term on the right-hand side of the above expression represents the manager's expected share of the firm's cash flows, and the second

[9] See, for example, Harris and Raviv (1989) and Innes (1990).

[10] See Nachman and Noe (1991) for a formalization of this argument and Innes (1990) for another argument in support of the monotonicity of contracts.

[11] The definition of $w(\cdot)$ off this interval is irrelevant. However, for definiteness, the definition of $w(\cdot)$ can be extended to the whole positive section of the real line by defining $w(x) = 0$ for x greater than $k(\bar{I})M$.

term represents the private benefits obtained by the manager. It follows that the participation constraints, which require that the manager's payoff from reporting truthfully be no less than her reservation utility, can be represented as

$$U^G(I_G, s_G) \geq U_0 \tag{PG}$$

and

$$U^B(I_B, s_B) \geq U_0, \tag{PB}$$

where (s_t, I_t), $t = B, G$, represents the contract offered to the manager in response to the report t. Expression (PG) represents the participation constraint for the manager conditional on signal G, and the second expression, (PB), represents the participation constraint for the manager conditional on the signal B.

The incentive compatibility constraints, which ensure that the manager truthfully reports her observed signal, are given by

$$U^G(I_G, s_G) \geq U^G(I_B, s_B) \tag{IG}$$

and

$$U^B(I_B, s_B) \geq U^B(I_G, s_G). \tag{IB}$$

The first of the above expressions, (IG), ensures that if the manager observes G, she is not better off reporting B than if she reports G, and the second expression, (IB), ensures that if she observes B, the manager is not better off reporting G than if she reports B.

Let $\mathcal{F}$ represent the set of sharing rule–investment pairs, $((s_G, I_G), (s_B, I_B))$, such that $(s_t, I_t) \in \mathcal{S} \times [0, \bar{I}]$, $t = G, B$, and (IG), (IB), (PG), and (PB) are satisfied. The firm's payoff, if it hires a manager who observes signal t and provides her with a contract (s, I), is the net expected cash flow from the project less the manager's compensation. To simplify the exposition, let this payoff be represented by the function $V^t(I, s)$, where

$$V^t(I, s) \equiv \int_0^\infty (x - s)N(I, t, dx) - I.$$

It follows that the firm's problem reduces to one of selecting $((s_G, I_G), (s_B, I_B))$ to solve the revelation problem

$$\max \mu V^G(I_G, s_G) + (1 - \mu)V^B(I_B, s_B) \tag{RP}$$

s.t.

$$((s_B, I_B), (s_G, I_G)) \in \mathcal{F}.$$

The remainder of the paper is devoted to obtaining a solution to this problem. In the next section, we partially solve this problem by proving that optimal contractual forms are independent of the level of investment.

3. OPTIMAL CONTRACT DESIGN

This section is devoted to the determination the optimal contract design. This is an interesting problem in its own right, and it greatly facilitates the determination of the firm's optimal investment policy. The basic insight is that there exist optimal contracts whose functional forms are independent of corporate investment as well as the relationship between private benefits and the manager's private information.

The requisite characterization is provided by Proposition 1, which demonstrates that, when characterizing the optimal investment policy, attention can be restricted to contracts that provide the manager with an option payment contingent on the report G and a salary contingent on the report B. An option contract with exercise price e is denoted s_e, where $s_e(x) = (x - e)^+ \equiv \max\{x - e, 0\}$. Similarly, a salary contract with payment d is denoted s_d, where $s_d(x) = x \wedge d \equiv \min\{x, d\}$.

PROPOSITION 1. *A contract pair in which the manager receives salary d if she reports B and an option with exercise price e if she reports G solves the contract design problem; i.e., there exist scalars d and e such that*

$$\max_{d,e} \{\mu V^G(I_G, s_e) + (1 - \mu)V^B(I_B, s_d)/(s_e, I_G) \; \& \; (s_d, I_B) \in \mathcal{F}, d, e \geq 0\}$$

$$= \max_{s_B, s_G} \{\mu V^G(I_G, s_G) + (1 - \mu)V^B(I_B, s_B)/(s_t, I_t) \in \mathcal{F}, t = B, G\}.$$

This result is very intuitive. Given the manager's report, each contractual form places maximal weight on the cash flows that are most likely to be generated. This follows by noting that HRO ensures that, regardless of the level of investment and the nature of the private benefit, the incremental probability of a high cash flow is uniformly higher for the manager if she observes G.

While this result demonstrates that the contract design problem can be solved by a salary–option pair, it does not preclude the existence of solutions specifying other contractual forms. In order for the option–

salary pair to be the uniquely optimal contractual form, it is necessary that both the incentive constraints bind and the cash flows be strictly HRO. For example, if the incentive constraint (IB) does not bind, any contract that has a functional form "sufficiently similar" to a salary contract and satisfies the participation constraint (PB) with equality will form part of an optimal solution to (RP). Even when incentive constraints bind, if the hazard-rate ordering is not strict, there will exist contractual forms that solve (RP) and differ from the option–salary pair over cash flow realizations where hazard rates are the same across types.[12]

Our goal in the remainder of the paper is to characterize the firm's investment decision under endogenous contracting. From this perspective, the limitations of Proposition 1 are unimportant. As we show in the following corollary, an investment level is optimal for (RP) if and only if it is also optimal when contract designs are restricted to the option–salary pair. Thus, Proposition 1 simplifies the analysis of the firm's optimal investment decision because it enables us to concentrate our attention on a contract set that can be parameterized by scalars. When restricted to providing the manager with an option contract if she reports G and a salary contract if she reports B, the firm's problem reduces to selecting $(((x - e)^+, I_G), (x \wedge d, I_B))$ to solve the restricted revelation problem

$$\max_{d,e,I_B,I_G} \mu V^G(I_G, (x - e)^+) + (1 - \mu)V^B(I_B, x \wedge d) \tag{RRP}$$

s.t.

$$(((x - e)^+, I_G), (x \wedge d, I_B)) \in \mathcal{F}.$$

The equivalence between (RP) and (RRP) in the context of the investment decision is formalized in Corollary 1.

COROLLARY 1. *There exists* $((s_G^*, I_G^*), (s_B^*, I_B^*))$ *that solves* (RP) *if and only if there exist scalars d and e such that* $(((x - e)^+, I_G^*), (x \wedge d, I_B^*))$ *solves* (RRP).

This result implies that any investment levels in a solution to (RP) must also satisfy the necessary conditions that characterize investment levels in a solution to (RRP), a finite-dimensional optimization problem. Because the necessary conditions for solutions to (RRP) can be obtained using elementary Lagrange multiplier techniques, this greatly simplifies the subsequent analysis of the firm's investment decision.

[12] We thank an anonymous referee for helping us clarify our thinking on this issue.

4. THE INVESTMENT PROBLEM

Our analysis begins with a general characterization of the solution to the firm's investment problem, after which attention is restricted to three interesting special cases for which determinate results are obtainable: the manager's private information pertains solely to the firm's cash flows, her private information pertains solely to private benefits from employment with marginal private benefits strictly increasing in type, and her private information pertains to both output and private benefits with marginal private benefits strictly decreasing in type.

To establish a benchmark for evaluating the investment levels that solve the revelation problem, we now define the first-best allocation. Let the firm's mean cash flow, given investment level I and signal t, be represented by $m^t(I)$, where

$$m^t(I) \equiv \int_0^\infty xN(I, t, dx).$$

Now let I_t^{po}, represent the Pareto optimal level of investment given signal t, where

$$I_t^{\text{po}} \equiv \underset{I}{\operatorname{argmax}}\ \text{PO}(I, t),$$

and

$$\text{PO}(I, t) \equiv m^t(I) + p(I, t) - I.$$

To simplify the notation further, let $\phi^t(y, I)$ represent the expected value of the cash flows from an option contract with an exercise price y, given investment I and signal t, where

$$\phi^t(y\ I) \equiv \int_0^\infty (x - y)^+ N(I, t, dx).$$

Given these definitions, if she receives the signal t, the payoff to the manager from a salary contract with payment d can be expressed as $m^t(I) - \phi^t(d, I)$. Similarly, if she receives signal t, the payoff to the manager from an option contract with exercise price e can be represented as $\phi^t(e, I)$. Using this simplification, we can represent the manager's marginal rate of substitution between contract payoffs and investment in a concise form. This trade-off between investment and contract value plays an important role in the characterization of optimal investment policies.

To formalize these notions, let $D_y\phi^t(y, I)$ and $D_I\phi^t(y, I)$ represent the derivatives of the function $\phi^t(y, I)$ with respect to $y = e$ or d and I, respectively. Similarly, let $D_I m^t(I)$ represent the derivative of the function $m^t(I)$ with respect to I. It follows that, if she observes the signal t, the manager's marginal rate of substitution between the exercise price of the option contract and investment level can be represented by $\mathrm{MRS}^t(e, I)$, where

$$\mathrm{MRS}^t(e, I) \equiv -\frac{D_I\phi^t(e, I) + D_I p(I, t)}{D_e\phi^t(e, I)}.$$

Similarly, if she observes the signal t, let the manager's marginal rate of substitution between the salary and investment level can be represented by $\mathrm{MRS}^t(d, I)$, where

$$\mathrm{MRS}^t(d, I) \equiv \frac{D_I m^t(I) - D_I\phi^t(d, I) + D_I p(I, t)}{D_d\phi^t(d, I)}.$$

The following proposition demonstrates that these marginal rates of substitution are crucial in determining the distortion in investment relative to the Pareto optimal.

PROPOSITION 2. *Given the report G, the firm will overinvest (underinvest) if and only if* $\mathrm{MRS}^G(e, I_G) - \mathrm{MRS}^B(e, I_G) > (<) 0$, *and the incentive constraint* (IB) *is binding. Similarly, given the report B, the firm will overinvest (underinvest), if and only if* (IG) *is binding and* $\mathrm{MRS}^G(d, I_B) - \mathrm{MRS}^B(d, I_B) > (<) 0$.[13]

The intuition underlying Proposition 2 is straightforward. Deviations from the Pareto optimal investment levels, if they occur, result from attempts to mitigate misreporting incentives while leaving participation constraints satisfied. If $\mathrm{MRS}^G(e, I_G) > (<) \mathrm{MRS}^B(e, I_G)$, by increasing (decreasing) both investment and exercise price, it is possible to leave the manager's utility, contingent on receiving signal G, unchanged while lowering her signal B-contingent utility. In this case, overinvestment (underinvestment) will mitigate misrepresentation incentives associated with

[13] In some cases $\mathrm{MRS}^t(d, I_B)$ is not defined, because the denominator of the expression, $D_d\phi^t(d, I_B)$, may be zero. This occurs when the salary is so high that all the firm's cash flows go to the manager. This case appears to be pathological. However, even in this case $\mathrm{MRS}^t(d, I_B)$ will be defined in some neighborhood of this singular point. Since the singular point has measure zero, if we integrate along the indifference curve of the manager, we see that the direction of overinvestment or underinvestment will depend upon the relative magnitudes of the two MRSs in the neighborhood of the singular point. Therefore, results derived below also obtain in this case.

the signal B. Similarly, the firm will overinvest (underinvest) in response to the report B when $\mathrm{MRS}^G(d, I_B) - \mathrm{MRS}^B(d, I_B) > (<) 0$. Here misrepresentation incentives are reduced by increasing (decreasing) investment expenditure and decreasing (increasing) the manager's salary as, by doing so, it is possible to leave the manager's signal B-contingent utility unaffected while lowering the utility associated with signal G.

The determination of the solution to the firm's investment problem is crucially dependent on the identity of the binding constraints, which is, in turn, dependent on the relationship between the contract–investment marginal rates of substitution across types. The nature of this relationship depends on the relationship between private benefits and type. Lemma 1 simplifies the analysis by demonstrating that the incentive constraint (IG) is always satisfied with equality when the other constraints are satisfied and binds when private benefits are strictly increasing in type. Thus, it establishes that investment distortions are more likely to occur in response to the report B.

LEMMA 1. (i) *If* $(((x - e)^+, I_G), (x \wedge d, I_B))$ *is a feasible solution to* (RP), (IG) *is satisfied with equality.* (ii) *If* $p(I, G) > p(I, B)$ *for all* $I \in [0, \bar{I}]$ *and* $(((x - e)^+, I_G), (x \wedge d, I_B))$ *is a feasible solution to* (RP), (IG) *binds.*

The intuition behind Lemma 1 is quite simple. If the incentive constraint (IG) is satisfied as a strict inequality and private benefits are increasing in type, because the manager obtains larger private benefits and output is higher if she observes G, the participation constraint (PG) is satisfied as a strict inequality. As neither (IG) nor (PG) is binding, the payment to the manager, contingent on the report G, can be reduced. This increases the value of the firm, demonstrating that the incentive constraint (IG) must be satisfied with equality in any solution to (RRP) and must bind when private benefits are strictly increasing in type.

In some cases, the solution to (RRP) can be further simplified. In one such case, when the manager does not obtain any private benefits, she may have an incentive to misreport both signals. Here, as is demonstrated in Corollary 2, when the disturbance terms are strictly hazard-rate ordered, only (IB) binds and $\mathrm{MRS}^G(d, I) - \mathrm{MRS}^B(d, I) > 0$. It follows that increased investment in response to the report B reduces misreporting incentives if the manager observes G. Further, because (IB) does not bind, the firm will invest at the Pareto optimal level in response to the report G.

COROLLARY 2. *When the manager does not obtain private benefits and the disturbance terms are strictly* HRO, *the firm will overinvest in response to report B and invest at the Pareto optimal level in response to the report G. This implies that for every* (I_G, I_B) *pair which is part of a* *solution to* (RP), $I_G = I_G^{\text{po}}$ *and* $I_B > I_B^{\text{po}}$. *Further, the value of compensation will be higher contingent on the report G.*

When the firm's cash flows are insensitive to the manager's private information and marginal private benefits are strictly increasing in type, the relationship between the first-best and the second-best investment levels is quite different. From Lemma 1 it follows that (IG) will bind. Since (IG) binds and increased investment by the firm in response to the report B increases the private benefit-related misrepresentation incentives of the manager if she observes G, the firm will underinvest. In this case, because contract values are insensitive to type, (IB) does not bind. Thus, the firm will invest the Pareto optimal amount in response to the report G. This result is proved in Corollary 3.

COROLLARY 3. *When the firm's cash flows are independent of the manager's private information,* $p(I, G) > p(I, B)$, *for all* I, *and* $D_I p(I, G) > D_I p(I, B)$, *the firm will invest at the Pareto optimal level when the manager reports G. On the other hand, the firm will always underinvest in response to report B; i.e., for every* (I_G, I_B) *pair which is part of a solution to* (RP), $I_G = I_G^{\text{po}}$, *and* $I_B < I_B^{\text{po}}$. *Further, the value of compensation is higher contingent on the report B.*

Thus far we have characterized the firm's investment decision in the extreme cases where only cash flow-related considerations or only private benefit-related considerations are important. The latter result was derived under the assumption that marginal private benefits are increasing in type. Under this assumption, distortions from Pareto optimal investment levels in intermediate cases, when both cash flow and private benefit considerations are important, will be determined by the relative importance of these considerations.

Now consider the case where private benefits are increasing in type, but marginal private benefits are decreasing in type; i.e., $p(I, G) > p(I, B)$ for all I, but $D_I p(I, G) < D_I p(I, B)$.[14] Here, in general, the manager will be required to overinvest if she reports B. Contingent on the observation of B by the manager, the direction of the deviation from the Pareto optimal investment level can be determined since both the private benefit component and the cash flow component of $\text{MRS}^G(I_B, d)$ are larger than the corresponding components for $\text{MRS}^B(I_B, d)$. Thus, increased investment minimizes the incentive to falsely report signal B.

On the other hand, when the Pareto optimal investment level is higher for type B, HRO ensures that (IB) will not bind. This follows because at the higher Pareto optimal investment level, private benefits provide no

[14] In the following section, we discuss conditions under which this parameterization of the relationship between private benefits and information is appropriate.

incentive for B to mimic G. At the same time the greater type sensitivity of the option contract eliminates any compensation-based misrepresentation incentives. Thus, in this case, no investment distortion results when the manager reports G. These results are provided in the following corollary.

COROLLARY 4. *When $p(I, G) > p(I, B)$ and $D_I p(I, G) < D_I p(I, B)$ for all $I \in [0, \bar{I}]$, for every I_B and I_G which are part of a solution to* (RRP), (i) *the manager will be required to overinvest if she reports B, and* (ii) *if she reports G, the manager will be required to invest the Pareto optimal amount whenever $I_B^{po} \geq I_G^{po}$. In this case, both marginal output and the value of compensation will be higher contingent on the report G.*

From the above discussion it is apparent that the primary determinants of the deviation from the Pareto optimal level of investment are the focus of the manager's private information and the sensitivity of the marginal private benefit to type. When the manager's private information relates primarily to firm output, the manager may have an incentive to misreport and can be dissuaded from doing so by raising the level of investment. On the other hand, when the manager's private information is focused primarily on private benefit generation, it becomes difficult for the manager to misreport if she observes B. Therefore, the firm will invest the Pareto optimal amount in response to the report G. By the same token, it is easier for the manager to misreport if she observes G. When the signal G is associated with a higher marginal private benefit, underinvestment in response to the report B discourages the manager from misrepresenting her information after observing G. On the other hand, when the marginal private benefit is decreasing in type, increased investment reduces both the contract-related and the private benefit-related incentives to misreport B. Therefore, in this case, the manager is required to overinvest after reporting B.

5. CONCLUSION

In this paper we have modeled the under- and overinvestment incentives engendered by private information regarding future prospects. In a framework which endogenizes the functional form of the managerial compensation contract, we have demonstrated that when managers possess private information primarily relating to future firm output, firms will overinvest. On the other hand, when managers' private information pertains primarily to their private benefits from investment, firms may underinvest.

These results have been derived using a multiplicative parameterization of uncertainty regarding output. However, our results are readily extendible to more general parameterizations. The contract design results in Proposition 1 extend to parameterizations of the investment–uncertainty relationship for which the hazard-rate ordering is satisfied for all levels of investment.[15] Our investment-related results are somewhat more sensitive to changes in the specification of output uncertainty. However, as can be seen from Proposition 2, any specification of the relationship between investment and output uncertainty that maintains the ordering of the marginal rates of substitution between information signals will generate the same investment comparative statics. Heuristically, this requires that increases in the level of investment result in increased uncertainty regarding output, which seems to be a natural assumption. However, if uncertainty regarding output does not increase with investment, for example, if $\tilde{X}_t = k(I) + \tilde{Z}_t$, no investment distortion occurs, and if output uncertainty decreases with investment, for example, if $\tilde{X}_t = k(I) + \tilde{Z}_t/I$, our results are reversed, in that differential information regarding firm output provides incentives for underinvestment.

Our model also generates a number of empirical hypotheses. For industries in which the private benefit component of returns is not significant, Corollary 2 predicts that overinvestment will be observed. Overinvestment implies a marginal return on investment lower than the cost of capital. Thus, Corollary 2 predicts that *in industries in which private benefits are an insignificant component of investment returns, the marginal return on capital investment will be below the required rate of return*. As the marginal returns can be estimated (see, for example, Antle, 1984; Lau, 1978) the hypothesis is directly testable. Corollary 2 also predicts that *more upside-sensitive compensation packages will have higher values*.[16]

In industries in which uncertainty regarding private benefits from investment is the focus of the agency problem, i.e., private-benefit-rich industries, there exists a severe estimation problem in inferring the direction of derivations from the Pareto optimal level of investment because marginal private benefits are not observable. In this case, observed returns on investment will tend to understate the true return. Thus, in these industries, standard econometric estimates of the return on capital may indicate overinvestment even though firms are underinvesting.

While this estimation problem makes it difficult to test our hypothesis regarding the direction of investment distortion, Corollary 3 permits us to predict the relationship between investment and management compensa-

[15] As can be seen by examining the Proof of Proposition 1, the hazard-rate ordering is required for the universal optimality of the salary–option pair.

[16] Upside and downside risk can be differentiated using Lower Partial Moments (see, for example, Bawa and Lindenberg, 1977).

tion for private-benefit-rich industries. For example, it supports the hypothesis that *higher levels of investment will be associated with lower levels of management compensation.* We are also able to obtain empirically testable hypotheses in the case where the locus of uncertainty is both private benefits and cash flows, the negative relationship between marginal private benefits and firm quality is sufficiently strong, and the salary contract is relatively riskless. In this case, Corollary 4 shows that *higher marginal returns to investment will be associated with more generous and more upside-sensitive compensation.*[17]

What sort of industries fit our model? The durable goods industry, in which there exists considerable uncertainty regarding short-run demand for a firm's product, while uncertainty regarding the private benefits that managers obtain from employment is relatively small, fits the paradigm of an industry in which private benefits are insignificant and the conditions of Corollary 2 apply. On the other hand, the biotechnology industry appears to be characterized by extreme uncertainty regarding future growth prospects which could augment a manager's human capital. Thus, it fits the profile of a private-benefit-rich industry assumed in Corollary 3. The same dynamic appears to be present in the advertising industry, or any industry with low capital requirements, in which managers can use the increased visibility resulting from larger firm size to launch rival firms. A different dynamic seems to be present in cash-rich industries such as the oil industry, in which managers can earn private benefits from investments that entrench them. In this case, better managers, who have better employment options outside their firms, will attach lower value to entrenchment through investment. Thus, in this case, we would expect that the private benefits from investment will be negatively related to the managers private information as is posited in Corollary 4.

A number of interesting issues, which could be addressed by extending our model, have not been addressed in our analysis. For example, we might consider the financing decisions associated with these investments. These decisions could include the mix between inside and outside financing, the functional form of the financing instrument, and the choice between bank and public financing.

APPENDIX: PROOFS OF PROPOSITIONS

Outline of Proof of Proposition 1. First note that since the set of salary and option contracts is a subset of the feasible set, $\mathcal{S}$, it follows that

[17] We are indebted to an anonymous referee for focusing our attention on these hypotheses.

$$\sup_{s_B, s_G} \{\mu V^G(I_G, s_G) + (1 - \mu)V^B(I_B, s_B) \mid ((s_G, I_G), (s_B, I_B)) \in \mathcal{F}\}$$

$$\geq \sup_{d \geq 0, e \geq 0} \{\mu V^G(I_G, (x - e)^+)$$

$$+ (1 - \mu)V^B(I_B, x \wedge d) \mid (((x - e)^+, I_G), (x \wedge d, I_B)) \in \mathcal{F}\}.$$

Note also that, since $\mathcal{S}$ is compact, as is the interval $[0, \bar{I}]$, the firm's choice set, $\mathcal{S} \times \mathcal{S} \times [0, \bar{I}] \times [0, \bar{I}]$, is also compact (see Tychonoff's Theorem, Munkres, 1975, pp. 232). Further, the valuation functions V^t: $\mathcal{S} \times [0, \bar{I}] \rightarrow \mathbb{R}$ are continuous, and (F) implies that the feasible set is nonempty. From this we know that the supremum on the left-hand side of the above inequality is obtained. The desired result follows if we can show that whenever (s_t^*, I_t^*), $t = G, B$, maximize the expected value of the firm, then there exist e and d such that

$$\mu V^G(I_G^*, s_G^*) + (1 - \mu)V^B(I_B^*, s_B^*) \tag{DE}$$

$$= \mu V^G(I_G^*, (x - e)^+) + (1 - \mu)V^B(I_B^*, x \wedge d).$$

Although a complete proof of this result is somewhat tedious, the idea behind the proof of this result is transparent. For the sake of clarity the proof is presented in its entirety. To begin, we show that, for a given contract value contingent on the signal B, a salary contract minimizes the payoff contingent on signal G. This is established in Lemma A1. Similarly, for a fixed contract value contingent on signal G, an option contract minimizes the payoff contingent on signal B. This is established in Lemma A2, the proof of which is already established in Nachman and Noe (1993). Using Lemmas A1 and A2, it easily follows that if we substitute for the optimal contracts, a debt (an option) contract, with the same type-contingent value contingent on the report B (G), the incentive and participation constraints will continue to be satisfied, and the firm's expected payoff will remain unchanged.

LEMMA A.1. *Suppose that $\int x\, B(dx) \geq a$, $G(x)$ dominates $B(x)$ in HRO, and the supports of $B(\cdot)$ and $G(\cdot)$ are compact and contained in $[0, \infty)$.*[18] *Define d as*

$$d = \inf \{y \in \mathbb{R} \mid \int (x \wedge y) B(dx) = a\}.$$

Then $x \wedge d$ solves

[18] For ease of exposition, in many cases, the limits of integration are not indicated. In these cases, integration is performed over the interval $[0, \infty)$. Because the supports of the distributions of the random variables are assumed to be compact, integration by parts is valid.

$$\min \int s \, G(dx) \tag{P}$$

s.t.

$$s \in \mathcal{S}, \qquad \int s \, B(dx) = a.$$

Proof. It is clear that the constraint $\int s\, B(dx) = a$ is binding only from above. Hence, we can replace the constraint $\int s\, B(dx) = a$ with $\int s\, B(dx) \geq a$ without altering the solution to the problem. After doing this and integrating by parts, we obtain

$$\min \int w(x)(1 - G(x))\, dx \tag{P$'$}$$

s.t.

$$0 \leq w(x) \leq 1, \int w(x)(1 - B(x))\, dx \geq a.$$

Forming the Lagrange for this problem, we obtain

$$L(w, \zeta) = \int w(x)[(1 - G(x)) - \zeta(1 - B(x)))]\, dx + \zeta a,$$

or equivalently

$$L(w, \zeta) = \int w(x)(1 - B(x)) \left[\frac{1 - G(x)}{1 - B(x)} - \zeta\right] dx + \zeta a.$$

Let $\zeta^* = (1 - G(d))/(1 - B(d))$. Then, HRO implies that the ratio $(1 - G(x))/(1 - B(x))$ is increasing in x, and hence, $(1 - G(x))/(1 - B(x)) - \zeta^* \leq 0$ whenever $x \leq d$, and $(1 - G(x))/(1 - B(x)) - \zeta^* \geq 0$ whenever $x \geq d$. It follows that, if we let $w^*(x) = 1$ if $x \leq d$ and $w^*(x) = 0$ if $x > d$, we see that

$$\begin{aligned} \inf_w L(w, \zeta^*) &= \int w^*(x)[1 - G(x) - \zeta^*(1 - B(x))]\, dx + \zeta^* a \\ &= \int (x \wedge d) G(dx). \end{aligned}$$

By the weak duality theorem we know that $\inf_w L(w, \zeta^*) = L(w^*, \zeta^*) \leq \inf(\text{P})$ (see Luenberger, 1969, p. 225). On the other hand, by the definition of d, we know that the indefinite integral of $w^*(x)$ (which equals $x \wedge d$) is feasible for (P). Hence,

$$L(w^*, \zeta^*) = \int (x \wedge d) G(dx) = \inf(\text{P}).$$

■

LEMMA A.2. *Suppose that* $\int x\, G(dx) \geq a$, $G(x)$ *dominates* $B(x)$ *in HRO, and the supports of* $B(\cdot)$ *and* $G(\cdot)$ *are compact and contained in* $[0, \infty)$. *Define e as*

$$e = \inf \{y \in \mathbb{R} \mid \int (x - y)^+ G(dx) = a\}.$$

Then $(x - e)^+$ *solves*

$$\min \int s\, B(dx) \tag{P*}$$

s.t.

$$s \in \mathcal{S}, \qquad \int s\, G(dx) = a.$$

Proof. It is clear that the constraint $\int s\, G(dx) = a$ is binding only from above. Hence, we can replace the constraint $\int s\, G(dx) = a$ with $\int s\, G(dx) \geq a$ without altering the solution to the problem. After doing this and integrating by parts, we obtain

$$\min \int w(x)(1 - B(x))\, dx \tag{P*$'$}$$

s.t.

$$0 \leq w \leq 1, \qquad \int w(x)(1 - G(x))\, dx \geq a.$$

The Lagrange for this problem can be represented as

$$L(w, \zeta) = \int w(x)[1 - B(x) - \zeta(1 - G(x)))]\, dx + \zeta a,$$

or equivalently

$$L(w, \zeta) = \int w(x)(1 - G(x)) \left[\frac{1 - B(x)}{1 - G(x)} - \zeta\right] dx + \zeta a.$$

Let $\zeta^* = (1 - B(e))/(1 - G(e))$. The HRO assumption implies that $(1 - B(x))/(1 - G(x))$ is decreasing in x, and hence, $(1 - B(x))/(1 - G(x)) - \zeta^* \geq 0$ whenever $x \leq e$, and $(1 - B(x))/(1 - G(x)) - \zeta^* \leq 0$ whenever $x \geq e$. Therefore, if we let $w^*(x) = 0$, if $x \leq e$ and $w^*(x) = 1$, if $x > e$, we see that

$$\begin{aligned}\inf_w L(w, \zeta^*) &= \int w^*(x)[1 - B(x) - \zeta^*(1 - G(x))]\, dx + \zeta^* a \\ &= \int (x - e)^+ B(dx).\end{aligned}$$

By the weak duality theorem we know that $\inf_w L(w, \zeta^*) = L(w^*, \zeta^*) \leq \inf(\mathrm{P}^*)$. On the other hand, by the definition of e, we know that the indefinite integral of $w^*(x)$ (which equals $(x - e)^+$) is feasible for (P^*). It follows that

$$L(w^*, \zeta^*) = \int (x - e)^+ B(dx) = \inf(\mathrm{P}^*).$$

■

Proof of Proposition 1. Let (s_G^*, s_B^*) be such a solution to (RP). To conclude the proof of Proposition 1, we need only establish the desired equality (DE). Let

$$a_G = \int s_G^* N(I_G^*, G, dx); \qquad a_B = \int s_B^* N(I_B^*, B, dx);$$
$$\mathcal{S}_B^* = \left\{ s \in \mathcal{S} \,\middle|\, \int s\, N(I_B^*, B, dx) = a_B \right\};$$

and

$$\mathcal{S}_G^* = \left\{ s \in \mathcal{S} \,\middle|\, \int s\, N(I_G^*, G, dx) = a_G \right\}.$$

Define

$$d = \inf \{ y \in \mathbb{R} \mid \int (x \wedge y) N(I_B^*, B, dx) = a_B \},$$

and

$$e = \inf \{ y \in \mathbb{R} \mid \int (x - y)^+ N(I_G^*, G, dx) = a_G \}.$$

Note that all contract pairs $(s_G, s_B) \in (\mathcal{S}_B^*, \mathcal{S}_G^*)$ produce the same value for the objective function of (RP). Thus, to show that $x \wedge d$ and $(x - e)^+$ solve (RP), we need only show that they are feasible. However, this is straightforward. Note that because $x \wedge d \in \mathcal{S}_B^*$,

$$\int s_B^* N(I_B^*, B, dx) = \int (x \wedge d) N(I_B^*, B, dx). \tag{A1}$$

Because $(x - e)^+ \in \mathcal{S}_G^*$,

$$\int s_G^* N(I_G^*, G, dx) = \int (x - e)^+ N(I_G^*, G, dx). \tag{A2}$$

Because $x \wedge d \in \mathcal{S}_B^*$ and $N(I_B^*, G, x)$ dominates $N(I_B^*, B, x)$ by HRO, Lemma A1 implies that

$$\int s_B^* N(I_B^*, G, dx) \geq \int (x \wedge d) N(I_B^*, G, dx). \tag{A3}$$

Because $(x - e)^+ \in \mathcal{S}_G^*$, and $N(I_G^*, G, x)$ dominates $N(I_G^*, B, x)$ by HRO, Lemma A2 implies that

$$\int s_G^* N(I_G^*, B, dx) \geq \int (x - e)^+ N(I_G^*, B, dx). \qquad \text{(A4)}$$

Equations (A1) and (A2) imply that $(x - e)^+$ and $x \wedge d$ provide the firm with the same payoff as s_G^* and s_B^*. The fact that the private benefits component, $p(\cdot)$, is the same under $(x - e)^+$ $(x \wedge d)$ and $s_G^*(s_B)$, combined with (A1) and (A2), demonstrates that the option contract and the salary contracts satisfy (PG) and (PB), respectively. Because the private benefit component of the managers' payoffs are the same under $(x - e)^+(x \wedge d)$ and $s_G^*(s_B^*)$, expressions (A2) and (A3) show that (IG) is satisfied by $((x - e)^+, x \wedge d)$, and (A1) and (A4) show that (IB) is also satisfied by $((x - e)^+, x \wedge d)$. ■

Proof of Corollary 1. First we show that if $((s_G^*, I_G^*), (s_B^*, I_B^*))$ solves (RP), then there exist scalars d and e such that $(((x - e)^+, I_G^*), (x \wedge d, I_B^*))$ solves (RRP). Next we show that if $(((x - e)^+, I_G^*), (x \wedge d, I_B^*))$ solves (RRP), then there exist contracts s_G^* and s_B^* such that $((s_G^*, I_G^*), (s_B^*, I_B^*))$ solves (RP).

Suppose that $((s_G^*, I_G^*), (s_B^*, I_B^*))$ solves (RP); then from Proposition 1 it follows that there exist scalars d and e such that $(((x - e)^+, I_G^*), (x \wedge d, I_B^*)) \in \mathcal{F}$ and $V^G(I_G^*, s_G^*) = V^G(I_G^*, (x - e)^+)$ and $V^B(I_B^*, s_B^*) = V^B(I_B^*, x \wedge d)$. It follows that the contracts $(((x - e)^+, I_G^*), (x \wedge d, I_B^*))$ solve (RP). Note that $(((x - e)^+, I_G^*), (x \wedge d, I_B^*))$ is a feasible solution to (RRP). Because the choice set for (RRP) is smaller than the choice set for (RP), the principal's maximum payoff attainable from (RP) cannot be smaller than the payoff attainable under (RRP). Thus, $(((x - e)^+, I_G^*), (x \wedge d, I_B^*))$ must be a solution to (RRP).

Now we show that if there exist scalars d and e such that $(((x - e)^+, I_G^*), (x \wedge d, I_B^*))$ solves (RRP), then there exist contracts s_G^* and $_B^*$ such that $((s_G^*, I_G^*), (s_B^*, I_B^*))$ solves (RP). We know that there exists a solution to (RP). Let $((s_G', I_G'), (s_B', I_B'))$ be any solution to (RP). From our earlier argument, there exist scalars d' and e' such that $(((x - e')^+, I_G'), (x \wedge d, I_B'))$ solves (RRP). By hypothesis, $(((x - e)^+, I_G^*), (x \wedge d, I_B^*))$ solves (RRP). Thus, the payoff to the principal from $(((x - e)^+, I_G^*), (x \wedge d, I_B^*))$ must equal the principal's payoff from $(((x - e')^+, I_G'), (x \wedge d', I_B'))$. But this implies that the principal's payoff from $(((x - e)^+, I_G^*), (x \wedge d, I_B^*))$ must be the same as his payoff from $((s_G', I_G'), (s_B', I_B'))$. The proof is concluded by setting $s_G^*(x) = (x - e)^+$ and $s_B^*(x) = x \wedge d$. ■

Proof of Proposition 2. The Lagrangean for the firm's problem (RRP) can be represented as

$$\begin{aligned} L(I_G, I_B, e, d, \lambda) & \\ \equiv \mu\{m^G(I_G) - \phi^G(e, I_G) - I_G\} &+ (1 - \mu)\{\phi^B(d, I_B) - I_B\} \quad \text{(SRP)} \\ & - \lambda_1\{U_0 - \phi^G(e, I_G) - p(I_G, G)\} \\ & - \lambda_2\{U_0 - m^B(I_B) + \phi^B(d, I_B) - p(I_B, B)\} \\ & - \lambda_3\{m^G(I_B) - \phi^G(d, I_B) + p(I_B, G) - \phi^G(e, I_G) - p(I_G, G)\} \\ & - \lambda_4\{\phi^B(e, I_G) + p(I_G, B) - m^B(I_B) + \phi^B(d, I_B) - p(I_B, B)\}, \end{aligned}$$

where $\lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$. Let $D_I\text{PO}(I, t)$ represent the derivative of the function $\text{PO}(I, t)$ with respect to I. It follows that if d, e, I_G, and I_B solve (SRP), then there exist $\lambda_j \geq 0, j = 1, 2, 3, 4$, such that

$$(\lambda_3 + \lambda_1 - \mu)\frac{D_e\phi^G(e, I_G)}{D_e\phi^B(e, I_G)} - \lambda_4 = 0, \tag{A5}$$

$$(1 - \mu - \lambda_2 - \lambda_4)\frac{D_d\phi^B(d, I_B)}{D_d\phi^G(d, I_B)} + \lambda_3 = 0, \tag{A6}$$

$$\mu D_I\text{PO}(I_G, G) - (\lambda_1 + \lambda_3 - \mu)D_e\phi^G(e, I_G)\{\text{MRS}^G(e, I_G) - \text{MRS}^B(e, I_G)\} = 0, \tag{A7}$$

$$(1 - \mu)D_I\text{PO}(I_B, B) + (1 - \mu - \lambda_2 - \lambda_4)D_d\phi^B(d, I_B)\{\text{MRS}^G(d, I_B) - \text{MRS}^B(d, I_B)\} = 0. \tag{A8}$$

Note that if (IB) binds, λ_4 is positive. It follows from (A5) that when λ_4 is positive, then $(\lambda_1 + \lambda_3 - \mu) > 0$. Also note that the term $D_e\phi^G(e, I_G)$ is negative. It follows that if $\text{MRS}^G(e, I_G) - \text{MRS}^B(e, I_G) > 0$, then for (A7) to be satisfied, $D_I\text{PO}(I_G, G)$ would have to be negative, implying overinvestment relative to the Pareto optimal. On the other hand, if $\text{MRS}^G(e, I_G) - \text{MRS}^B(e, I_G) < 0$, then for (A7) to be satisfied, $D_I\text{PO}(I_G, G)$ will have to be positive, representing underinvestment. Next consider the investment level contingent on the report B. Note that if (IG) binds, then $\lambda_3 > 0$, and for (A6) to be satisfied, $(1 - \mu - \lambda_2 - \lambda_4) < 0$. The remainder of the argument follows from the relationship between the signs of $\text{MRS}^G(d, I_B) - \text{MRS}^B(d, I_B)$ and $D_I\text{PO}(I_B, B)$ for (A8) to be satisfied. ■

Proof of Lemma 1. First we prove (i), and then we prove (ii). To establish these results we first demonstrate that if $p(I, G) \geq p(I, B)$ and (IG) and (PB) are satisfied, then (PG) is also satisfied (Lemma A3).

LEMMA A.3. *If* $(((x - e)^+, I_G), (x \wedge d, I_B))$ *satisfies* (PB) *and* (IG) *and* $p(I, G) \geq p(I, B)$ *for all* $I \in [0, \bar{I}]$, *then* $((x - e)^+, I_G)$ *satisfies* (PG).

Proof. The proof follows by noting that if the pair $(x \wedge d, I_B)$ satisfies (PB) then $U^B(I_B, x \wedge d) \geq U_0$. Since the contract $x \wedge d$ is monotone increasing, by FSD it follows that $m^G(I_B) - \phi^G(d, I_B) \geq m^B(I_B) - \phi^B(d, I_B)$ for all I. Because $p(I, G) \geq p(I, B)$ for all I, $U^G(I_B, x \wedge d) \geq U^B(I_B, x \wedge d)$. Note that if $(((x-e)^+, I_G), (x \wedge d, I_B))$ satisfies (IG), then $U^G(I_G, (x-e)^+) \geq U^G(I_B, x \wedge d)$. Thus, $U^G(I_G, (x-e)^+) \geq U^G(I_B, x \wedge d) \geq U^B(I_B, x \wedge d) \geq U_0$. ∎

Now we establish (i) by means of a contradiction. Suppose that (IG) is not satisfied with equality; i.e., $U^G(I_G, (x-e)^+) > U^G(I_B, x \wedge d)$. Because the solution is feasible, (PB) must hold; i.e., $U^B(I_B, x \wedge d) \geq U_0$. From FSD and $p(I, G) \geq p(I, B)$, it follows that $U^G(I_G, (x-e)^+) > U^G(I_B, x \wedge d) \geq U^B(I_B, x \wedge d) \geq U_0$. It follows that neither (IG) nor (PG) is satisfied with equality. Thus, we can reduce the manager's compensation until (IG) is satisfied as an equality. At this point, from Lemma A3, it follows that the firm's profit is higher and the other constraints (including (PG)) are satisfied. This is the desired contradiction.

Now we establish (ii). To see that (IG) must bind if $p(I, G) > p(I, B)$, note that the above arguments establish that if (IG) is satisfied as an equality, i.e., $U^G(I_G, (x-e)^+) = U^G(I_B, x \wedge d)$ and (PB) is satisfied, then from FSD and $p(I, G) > p(I, B)$ it must be the case that $U^G(I_G, (x-e)^+) = U^G(I_B, x \wedge d) > U^B(I_B, x \wedge d) \geq U_0$. Thus, (IG) must bind. ∎

LEMMA A4. *If d and e are scalars such that $(((x-e)^+, I_G), (x \wedge d, I_B))$ solves* (RRP), *then* (*strict*) *HRO implies that*

(i) $$\frac{D_I \phi^G(e, I_G)}{-D_e \phi^G(e, I_G)} \geq (>) \frac{D_I \phi^B(e, I_G)}{-D_e \phi^B(e, I_G)}, \tag{A9}$$

(ii) $$\frac{D_I m^G(I_B) - D_I \phi^G(d, I_B)}{D_d \phi^G(d, I_G)} \geq (>) \frac{D_I m^B(I_B) - D_I \phi^B(d, I_B)}{D_d \phi^B(d, I_B)}, \tag{A10}$$

(iii) $$r(d)(\phi^G(e, I_G) - [m^G(I_B) - \phi^G(d, I_B)]) \geq (>) \phi^B(e, I_G) - [m^B(I_B) - \phi^B(d, I_B)],$$

where $r(d) \leq (<) 1$.

Proof of Lemma A4. First we prove (i). Note that

$$\mathrm{MRS}^t(e, I_G) = -\frac{D_I \phi^t(e, I)}{D_e \phi^t(e, I)} = k'(I) \int_{e/k(I)}^{\infty} \frac{z\, dF_t(z)}{\bar{F}_t[e/k(I)]}.$$

Therefore, the sign of $\mathrm{MRS}^G(e, I_G) - \mathrm{MRS}^B(e, I_G)$ follows from the following general result: If $F_G(\cdot)$ has a smaller hazard rate than $F_B(\cdot)$, then, for all c,

$$\int_c^\infty \frac{z\, dF_G(z)}{\bar{F}_G(c)} \geq \int_c^\infty \frac{z\, dF_B(z)}{\bar{F}_B(c)}.$$

We prove this result as follows: Integrating each side of the above expression by parts, for $t = G, B$, we obtain

$$\int_c^\infty \frac{z\, dF_t(z)}{\bar{F}_t(c)} = c + \int_c^\infty \frac{\bar{F}_t(z)\, dz}{\bar{F}_t(c)}.$$

It follows that

$$\int_c^\infty \frac{z\, dF_G(z)}{\bar{F}_G(c)} - \int_c^\infty \frac{z\, dF_B(z)}{\bar{F}_B(c)} = \int_c^\infty \left\{\frac{\bar{F}_G(z)}{\bar{F}_G(c)} - \frac{\bar{F}_B(z)}{\bar{F}_B(c)}\right\} dz$$

$$= \int_c^\infty \frac{\bar{F}_G(z)}{\bar{F}_B(c)} \left\{\frac{\bar{F}_B(c)}{\bar{F}_G(c)} - \frac{\bar{F}_B(z)}{\bar{F}_G(z)}\right\} dz.$$

A sufficient condition for the above expression to be nonnegative (positive) is that the term in parentheses be nonnegative (positive). Given (strict) (HRO) and because $c \leq z$ this expression is nonnegative (positive). Letting $c = e/k(I)$, this proves (i). A similar argument can be used to prove (ii). Result (iii) is inequality (A3) in Nachman and Noe (1993) expressed in our notation. ■

Proof of Corollary 2. To prove this result we first show that (IG) binds. This proof is similar to that of Lemma 1(ii) and follows because (PG) does not bind when (PB), (IG), and (IB) are satisfied. Because (PG) does not bind, (IG) must bind. Given that (IG) binds, using Lemma A4(iii), we now show that (IB) does not bind. Note that because (IG) binds, $\phi^G(e, I_G) - [m^G(I_B) - \phi^G(d, I_B)] = 0$. Lemma A4(iii) implies then that $\phi^B(e, I_G) - [m^B(I_B) - \phi^B(d, I_B)] < 0$. That is, (IB) does not bind.

Note that if (IB) does not bind, $\lambda_4 = 0$. It follows from (A5) that $\lambda_1 + \lambda_3 = \mu$, implying that the manager will be required to invest at the Pareto optimal level if she reports G. Further, because (IG) binds and (from Lemma A4(ii)) $\mathrm{MRS}^G(d, I_B) > \mathrm{MRS}^B(d, I_B)$, from Proposition 2 it follows that the firm will overinvest in response to the report B.

To see that the value of compensation is higher contingent on the report G, note that since (IG) binds, $\phi^G(e, I_G) = m^G(I_B) - \phi^G(d, I_B)$. Further, from strict HRO, it follows that $m^G(I_B) - \phi^G(d, I_B) > m^B(I_B) - \phi^B(d, I_B)$. Thus, $\phi^G(e, I_G) > m^B(I_B) - \phi^B(d, I_B)$. ■

Proof of Corollary 3. We begin by demonstrating that the firm underinvests in response to the report B. First note that if the firm's cash flows are independent of the manager's private information,

$$m^B(I_B) - \phi^B(d, I_B) = m^G(I_B) - \phi^G(d, I_B)$$

and

$$\phi^B(e, I_G) = \phi^G(e, I_G). \tag{A11}$$

Since (IG) binds, by using (A11) to substitute for $D_I m^G(I_B) - D_I\phi^G(d, I_B)$ in (A8), it can be seen that $\mathrm{MRS}^G(d, I_B) < \mathrm{MRS}^B(d, I_B)$. This shows that the firm will underinvest in response to the report B.

Now we show that the firm will invest at the Pareto optimal level in response to the report G. To see this, note that, by using (A11), (IG) and (IB) can be rewritten as

$$m^B(I_B) - \phi^B(d, I_B) - \phi^B(e, I_G) = p(I_G, G) - p(I_B, G), \tag{IG*}$$

and

$$m^B(I_B) - \phi^B(d, I_B) - \phi^B(e, I_G) \geq p(I_G, B) - p(I_B, B), \tag{IB*}$$

It follows that, since $D_I p(I, G) > D_I p(I, B)$, (IB*) will be satisfied as a strict inequality whenever $I_G > I_B$. Since the firm underinvests in response to the report B, $I_B < I_B^{\mathrm{po}}$. Further, since firm output is insensitive to the manager's private information and $D_I p(I, G) > D_I p(I, B)$ for all I, it must be the case that $D_I\mathrm{PO}(I, G) > D_I\mathrm{PO}(I, B)$ for all I. This implies that $I_B^{\mathrm{po}} < I_G^{\mathrm{po}}$. It follows then that $I_B < I_B^{\mathrm{po}} < I_G^{\mathrm{po}}$. Consequently, (IB*) cannot bind if the firm invests at the Pareto optimal level in response to the report G. This proves the desired result.

To see that the value of compensation is higher under the report B, note that since $D_I m^G(I) = D_I m^B(I)$ and $D_I p(I, G) > D_I p(I, B)$ for all I, $D_I\mathrm{PO}(I, G) > D_I\mathrm{PO}(I, B)$ for all I. Thus, $I_G^{\mathrm{po}} \geq I_B^{\mathrm{po}}$. Further, from the above result, it follows that $I_B < I_B^{\mathrm{po}}$. Since (IG*) is satisfied, it must be the case that

$$m^B(I_B) - \phi^B(d, I_B) - \phi^G(e, I_G^{\mathrm{po}}) = p(I_G^{\mathrm{po}}, G) - p(I_B, G).$$

Since $p(I, G)$ is increasing in I and $I_G^{\mathrm{po}} > I_B^{\mathrm{po}}$, it must be the case that $p(I_G^{\mathrm{po}}, G) - p(I_B, G) > 0$. This implies that the left hand side of the above expression is positive. ■

Proof of Corollary 4. The first result follows directly from Lemma A4(ii) and by noting that, under the assumptions of this corollary, $D_I p(I, G) < D_I p(I, B)$. Since $D_d\phi^t(d, I_B) \leq 0$ and $D_d\phi^B(d, I_B) \geq D_d\phi^G(d, I_B)$, by adding $D_I p(I_B, G)$ and $D_I p(I_B, B)$ to the numerators of the MRS terms in (A10), we only increase the difference between them. This implies that, in this case, $\mathrm{MRS}^G(d, I_B) > \mathrm{MRS}^B(d, I_B)$. Further, from Lemma 1, it follows that (IG) binds. Therefore, it follows from Proposition 2 that the manager will be required to overinvest if she reports B.

We now establish the second claim by demonstrating that the constraint (IB) cannot bind. Consider the relaxed problem in which the constraint (IB) is not imposed. From an argument virtually identical to that used above, it follows that the solution to this problem will require that $I_B > I_B^{\text{po}}$. Further, given that (IB) is not imposed, the firm will invest I_G^{po} in response to the report G, and the salary, d, will be set so as to satisfy (PB). Since a solution to the relaxed problem, if it is feasible, must be a solution to the restricted problem (RRP), if we can show that this solution is feasible, we will have established its optimality. We prove this result by means of a contradiction. Suppose that (IB) is violated by this solution to the relaxed problem. Because (IG) binds, it must be the case that

$$m^G(I_B) - \phi^G(d, I_B) + p(I_B, G) = \phi^G(e, I_G^{\text{po}}) + p(I_G^{\text{po}}, G),$$

and

$$\phi^B(e, I_G^{\text{po}}) + p(I_G^{\text{po}}, B) > m^B(I_B) - \phi^B(d, I_B) + p(I_B, B).$$

Together, these expressions imply that

$$\begin{aligned} &[p(I_B, B) - p(I_G^{\text{po}}, B)] - [p(I_B, G) - p(I_G^{\text{po}}, G)] \\ &< \{\phi^B(e, I_G^{\text{po}}) - [m^B(I_B) - \phi^B(d, I_B)]\} - \{\phi^G(e, I_G^{\text{po}}) - [m^G(I_B) - \phi^G(d, I_B)]\}. \end{aligned}$$

Because $I_G^{\text{po}} \leq I_B^{\text{po}} < I_B$ and $D_I p(I, G) < D_I p(I, B)$ for all $I \in [0, \bar{I}]$, it follows that the left-hand side of the above inequality is strictly positive. Further, because $I_G^{\text{po}} < I_B$ and $p(\cdot)$ is increasing in I, $p(I_G^{\text{po}}, G) < p(I_B, G)$. Because $p(I_G^{\text{po}}, G) < p(I_B, G)$ and (IG) is satisfied as an equality, it must be the case that $\phi^G(e, I_G^{\text{po}}) > m^G(I_B) - \phi^G(d, I_B)$. It follows from Lemma A4(iii) that the right-hand side of the above inequality is nonpositive. This is the desired contradiction, implying that (IB) cannot bind.

To establish the result regarding the marginal rates of return, note that, HRO ensures that marginal returns to investment are lower for type B; i.e., $D_I m^G(I) \geq D_I m^B(I)$ for all I. Further, $k(I)$ is concave, implying that $D_I m^B(I_G^{\text{po}}) \geq D_I m^B(I)$ for all $I \geq I_G^{\text{po}}$. Thus, $D_I m^G(I_G^{\text{po}}) \geq D_I m^B(I_G^{\text{po}}) \geq D_I m^B(I_B^{\text{po}}) \geq D_I m^B(I_B)$.

To see that the value of compensation contingent on the report G is higher, note that since $I_B > I_G^{\text{po}}$ and $p(I, G)$ is increasing in I, it must be the case that $p(I_G^{\text{po}}, G) < p(I_B, G)$. Since (IG) binds, it must be the case that

$$p(I_B, G) - p(I_G^{\text{po}}, G) = \phi^G(e, I_G^{\text{po}}) - [m^G(I_B) - \phi^G(d, I_B)] > 0.$$

Further, from FSD it follows that

$$m^G(I_B) - \phi^G(d, I_B) \geq m^B(I_B) - \phi^B(d, I_B).$$

Thus, it must be the case that

$$\phi^G(e, I_G^{\text{po}}) > m^G(I_B) - \phi^G(d, I_B) \geq m^B(I_B) - \phi^B(d, I_B).$$

■

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